

Code No: 183BJ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, December – 2024/ January - 2025
MATHEMATICAL AND STATISTICAL FOUNDATIONS
(Common to CSE (AI&ML), (AI&DS), AI&ML)

Time: 3 Hours

Max. Marks: 60

Note: This question paper contains two parts A and B.

i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

- a) Define perfect number, write largest two digits perfect number. [1]
- b) Find the remainder in division of 41^{65} by 7. [1]
- c) Write the confidence interval for intercept of regression line $y = \beta_0 + \beta_1 x$. [1]
- d) Probability density function of a random variable X is $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$
 find distribution function $F(x)$ of X . [1]
- e) Write mean and variance of the standard normal distribution. [1]
- f) Define sampling distribution. [1]
- g) In how many ways the estimation can be done and what are they. [1]
- h) Define Type-II error. [1]
- i) Define regular Markov chain. [1]
- j) $\begin{bmatrix} x & y \\ y & 0.6 \end{bmatrix}$ is Transition Probability Matrix (TPM), find x, y . [1]

PART-B

(50 Marks)

- a) Using the Euclidean algorithm compute G.C.D of (252, 198) and find $x, y \in \mathbb{Z}$ such that $252x + 198y = \text{G.C.D of } (252, 198)$.
- b) Prove that G.C.D of $(2^a - 1, 2^b - 1)$ is $2^x - 1$, where $x = \text{GCD of positive integers } a \text{ and } b$. [5+5]

OR

- a) Prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7. [5+5]
- b) Find the inverse of $A = \begin{bmatrix} 2 & 5 & 6 \\ 2 & 0 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ (modulo 7). [5+5]

- 4.a) The following data represent the chemistry grades for a random sample of 12 freshmen at a certain college along with their scores on an intelligence test administered while they were still seniors in high school

Test Score(X)	65	50	55	65	55	70	65	70	55	70	50	55
Intelligence test (Y)	85	74	76	90	85	87	94	98	81	91	76	74

Compute correlation coefficient.

- b) Derive the variance of binomial distribution $b(x; n, p)$. [5+5]

OR

- 5.a) The heights of mothers and daughters are given in the following table. Obtain the equation of line of regression and estimate the height of daughter when the height of the mother is 64.5 inches.

Height of the mother(inches) (X)	62	63	64	64	65	66	68	70
Height of the daughter (inches) (Y)	64	65	61	69	67	68	71	65

- b) Probability distribution of a random variable X is $F(x) = \begin{cases} 10kx^2 & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$

i) Find k , ii) Find the cumulative distribution of X , iii) find the mean and variance of x .

[5+5]

- 6.a) The probability that a patient recovers from a delicate heart operation is 0.9. Out of the next 100 patients having this operation, what is the probability that between 84 and 95 inclusive survive. Assume normal distribution as a limiting case of binomial distribution.

- b) A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn with replacement from this population. Find the mean of the sampling distribution of means and standard deviation of the sampling distribution of means. [5+5]

OR

- 7.a) If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kgs, how many students have masses between 65 and 71 kgs.

- b) The television picture tubes of manufacturer A have a mean life time of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean life time of 6.0 years and a standard deviation of 0.8 year. What is the probability that a random sample of 36 tubes from manufacturer A will have a mean life time that is at least 1 year more than the mean life time of a sample of 49 tubes from manufacturer B?

[5+5]

- 8.a) Prove that sample variance is an unbiased estimator of the population variance.

- b) A random sample of 10 boys had the following I.Q.'s 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Find the reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. [5+5]

OR

9.a) Two types of new cars produced in U.S.A. are tested for petrol mileage. One sample consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variances as 2 and 1.5 respectively. Test whether there is any significance difference in the petrol consumption of these types of cars. Use 0.01 level of significance.

b) The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal. [5+5]

10. The transition probability matrix of a Markov chain with states A, B, C is

$$\begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.1 \\ 0.2 & 0.6 & 0.2 \end{bmatrix} \text{ and}$$

give the initial probabilities of states A, B, C is (0.3, 0.4, 0.3).

- Determine whether the transition probability matrix is ergodic or not,
- Draw transition diagram,
- Find the transition probability that state A to C after two time period.
- Calculate q_3 .
- Find the steady state probability. [2+2+2+2+2]

OR

11. In a certain market, there are three brands of lipsticks A, B and C. Given that a lady
- Last purchased lipstick of brand A, there is 20% chances that she would shift to brand B, and 10% chances to C.
 - Last purchased lipstick of brand B, there is 50% chances that she would shift to brand A, and 10% chances to C.
 - Last purchased lipstick of brand C, there is 60% chances that she would shift to brand A, and 20% chances to B.

Suppose the present market share of the three brands A, B and C is 60%, 30% and 10% respectively, using this information find

- Transition probability matrices,
- Draw transition diagram,
- The probability that a customer who is currently a brand A purchaser will purchase brand B after two time period.
- The market share for brand A, B, C after three periods from now.
- Find the steady state probability. [2+2+2+2+2]